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[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 916

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Unique Paper Code : 2352571101

Name of the Paper : DSC: Topics in Calculus

Name of the Course : B.A. / B.Sc. (Prog.) with
Mathematics as Non-Major/
Minor

Semester : I

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any Two parts from each question.
3. All questions carry equal marks.

1. (a) If $f(x) = \begin{cases} \frac{x}{1+e^{1/x}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

show that f is continuous but not differentiable at $x = 0$.

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(b) If $y = e^{\tan^{-1}x}$, prove that

$$(1 + x^2)y_{n+2} + \{2(n+1)x - 1\}y_{n+1} + n(n+1)y_n = 0$$

(c) State Euler's theorem and verify it for $z = \sin^{-1} \frac{x}{y} +$

$$\tan^{-1} \frac{y}{x}.$$

2. (a) If $f(x) = \begin{cases} \frac{x}{|x|}, & x \neq 0 \\ 0, & x = 0 \end{cases}$, check continuity of the function f at $x = 0$ and specify the type of discontinuity, if any.

(b) Find the n^{th} derivative of $y = \cos^2 x \sin^3 x$.

(c) If $u = \log \frac{x^4 + y^4}{x + y}$, show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 3$.

3. (a) State Lagrange's mean value theorem and use it to show that

$$1 + x < e^x < 1 + xe^x, \quad x > 0.$$

(b) Prove

$$\sin ax = ax - \frac{a^3 x^3}{3!} + \dots + \frac{a^{n-1} x^{n-1}}{(n-1)!}$$

$$\sin\left(\frac{(n-1)\pi}{2}\right) + \frac{a^n x^n}{n!} \sin\left(a\theta x + \frac{n\pi}{2}\right).$$

(c) Find a, b, c so that $\lim_{x \rightarrow 0} \frac{ae^x - b \cos x + c e^{-x}}{x \sin x} = 2$.

4. (a) Verify Rolle's theorem for

(i) $x^3 - 6x^2 + 11x - 6$, $x \in [1, 3]$

(ii) $\sin x$, $x \in [0, \pi]$

(b) State Taylor's theorem with Lagrange's form of remainder. Find the Taylor series expansion of $f(x) = \sin x$.

(c) Evaluate $\lim_{x \rightarrow 0} \frac{\tan x - x}{x^2 \tan x}$.

5. (a) Find all the asymptotes of the curve

$$x^2 y^2 (x^2 - y^2)^2 = (x^2 + y^2)^3.$$

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(b) Trace the curve

$$y^2(a + x) = x^2(a - x), \quad a > 0.$$

(c) Find a reduction formula for

$$\int \cos^n x dx.$$

Hence evaluate $\int_0^{\frac{\pi}{2}} \cos^5 x dx$.

6. (a) Determine the position and nature of double points on the curve

$$x^3 - y^2 + 2x^2 + 2xy + 5x - 2y = 0.$$

(b) Obtain a reduction formula for $\int \sin^m x \cos^n x dx$.

Hence evaluate $\int_0^{\frac{\pi}{2}} \sin^2 x \cos^3 x dx$.

(c) Trace the curve

$$x^2(a - x) = ay^2, \quad a > 0.$$